Late Time Neutrino Masses, the LSND Experiment and the Cosmic Microwave Background

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Models with low-scale breaking of global symmetries in the neutrino sector provide an alternative to the seesaw mechanism for understanding why neutrinos are light. Such models can easily incorporate light sterile neutrinos required by the LSND experiment. Furthermore, the constraints on the sterile neutrino properties from nucleosynthesis and large scale structure can be removed due to the non-conventional cosmological evolution of neutrino masses and densities. We present explicit, fully realistic supersymmetric models, and discuss the characteristic signatures predicted in the angular distributions of the cosmic microwave background.

Introduction — The LSND experiment found evidence for the oscillations $\bar{\nu}_\mu \to \bar{\nu}_e$ and $\nu_\mu \to \nu_e$ with an oscillation probability of around 3×10^{-3} [1] and a $\Delta m^2 \gtrsim 1\text{eV}^2$. The statistical evidence for the anti-neutrino oscillations is much stronger than for the neutrino case, with some analyses finding a 5σ effect [2]. While other experiments restrict the regions of parameter space that could explain the LSND data, they do not exclude the LSND result [3].

The confirmation of solar and atmospheric neutrino oscillations has led to a "standard" framework for neutrino masses, with three light neutrinos resulting from the seesaw mechanism and the heaviest right-handed neutrino not far from the scale of gauge coupling unification [4]. The LSND result conflicts with this framework, and, if confirmed by the Mini-BooNE experiment [5], would throw neutrino physics into a revolutionary phase. There are three major challenges to incorporate the LSND result into the standard framework for neutrino masses. A fourth light neutrino is needed with mass in the eV range, and, given the Z width, this neutrino must be sterile. Such states are anathema to the seesaw mechanism. Secondly, neutrino oscillations in the early universe would ensure that this fourth neutrino state is thermally populated during the big bang nucleosynthesis (BBN), changing the expansion rate and yielding light element abundances in disagreement with observations [6]. In fact, theories with four neutrino species give very poor fits to the combined data of the LSND experiment and oscillation experiments with null results, while fits in theories with 5 neutrinos are much better [7]. This would imply that $N_{\nu\mathrm{BBN}} \geq 5$, in gross disagreement with observations [8, 9]. Finally, the combination of large scale structure surveys and WMAP data [10, 11] has led to a limit on the sum of the neutrino masses of about 0.7 eV, which is significantly less than the best fit values for the LSND neutrino masses [9, 11].

An alternative explanation for why the neutrinos are light has been explored recently: the scale of neutrino masses can be dictated by a low scale f of breaking of global symmetries in the neutrino sector [12, 13]. A characteristic feature of this mechanism is that, cosmologically, neutrinos remain massless until the symmetry-breaking phase transition, which occurs quite late in the history of the universe — hence the name "late time neutrino masses". In this letter, we argue that, unlike the traditional seesaw framework, this alternative scenario can easily accommodate the sterile neutrinos required by the LSND experiment. Moreover, the cosmological evolution of neutrino masses and densities in this scenario is non-standard and, as a result, the apparent contradiction between the parameters preferred by LSND and cosmology can be avoided, provided that f is of order 100 keV. (We will show that such values of f can arise naturally in supersymmetric theories.) At the same time, the scenario predicts potentially observable characteristic signatures in the cosmic microwave background (CMB) angular distributions.

All of the above features are generic in models with low-scale breaking of neutrino global symmetries, and can be understood without reference to a specific model. Let us outline the basic physical arguments.

The traditional seesaw framework relies on the gauge quantum numbers of the neutrinos to explain their mass spectrum, and leads to a picture where only active neutrinos are light. Global symmetries, on the other hand, may involve sterile as well as active neutrinos, and they may forbid both Dirac and Majorana mass terms. Neutrino masses then appear as a result of spontaneous breaking of these symmetries, so that, in this scenario, it is quite natural to expect light sterile states.

If the symmetry breaking phase transition occurs *after* the BBN epoch, both active and sterile neutrinos are massless before and during nucleosynthesis. In this case, the oscillations, that typically lead to thermal abundances for the sterile states in the traditional scenario, are absent. During BBN, the energy density of the sterile neutrinos (and of the scalars required to break the global symmetries) is determined by their temperature. As we show below, this temperature can be significantly lower than that of the rest of the cosmic fluid, provided

that the rates of certain reactions are sufficiently low. This allows our models to evade the BBN constraints on ΔN_ν.

Remarkably, the limit from large scale structure on the sum of the neutrino masses is also easily avoided by the late time neutrino mass models. The breaking of the global symmetries gives rise to a set of Goldstone bosons, which are coupled to both active and sterile neutrinos. This coupling is sufficiently strong for the sterile neutrinos to disappear after they become non-relativistic, for example by decaying into an active neutrino and a Goldstone boson. As a result, the relic abundance of the sterile neutrinos is low, and they do not significantly contribute to dark matter despite their large mass.

Specific Models— There is a wide variety of late time neutrino mass models: the neutrinos may be either Dirac or Majorana, the number of sterile neutrinos may vary, and different choices of global symmetries and their breaking patterns can be made. Let us present two simple supersymmetric models incorporating the LSND neutrinos. We do not consider models with a single sterile neutrino, as they are disfavored by oscillation data. For concreteness we construct models with three mass eigenstates that are predominantly sterile. We take the global symmetry to be $U(1) \times U(1)$; a simple possibility that allows a heavy neutrino to decay to light neutrino and a Goldstone boson.

Our first theory has three right-handed neutrino superfields, n. There is no overall lepton number symmetry, leading to six physical Majorana neutrinos. Above the weak scale the theory is described by the superpotential

$$\begin{aligned} W^M &= W_{NMSSM} + W_\nu^M, \\ W_\nu^M &= \lambda_{ij} l_i n_j h \, \frac{\phi}{M} + \frac{\kappa}{3}\phi^3 + \tilde{\lambda}_{ij} n_i n_j s \, \frac{\tilde{\phi}}{M} + \frac{\tilde{\kappa}}{3}\tilde{\phi}^3, \quad (1) \end{aligned}$$

where W_{NMSSM} is the superpotential of the NMSSM; λ, $\tilde{\lambda}$, κ and $\tilde{\kappa}$ are coupling constants; and the flavor indices i and j run from $1 \to 3$. The superfields l, h are the lepton and Higgs doublets of the MSSM, s is the electroweak singlet field of the NMSSM, and ϕ, $\tilde{\phi}$ are the extra electroweak singlet fields whose vacuum expectation values (vevs) give masses to neutrinos. The non-renormalizable operators in (1) are generated by integrating out physics at scale M; phenomenological constraints discussed below imply $M \sim 10^9$ GeV. In theories without an s field, the third operator in W_ν^M would be absent, and we would expect three light Dirac neutrinos. (If $nn\tilde{\phi}$ were allowed, the sterile states would be much heavier than the active states.) However, for theories such as the NMSSM, where the s field acquires a vev of order the electroweak scale, the Dirac and Majorana mass terms are of the same order of magnitude, vf/M, explaining why the LSND neutrinos are quite close in mass to the active neutrinos. W^M is the most general superpotential in the neutrino sector up to dimension four under the following discrete symmetries: Z_3, under which all the fields except ϕ and $\tilde{\phi}$ have charge $2\pi/3$; Z_3', under which s, h and $\bar{h}$ are uncharged, q, l, n and $\phi, \tilde{\phi}$ have charge $2\pi/3$, while u^c, d^c and e^c have charge $-2\pi/3$; and Z_3'', under which n and $\tilde{\phi}$ both have charge $2\pi/3$ while ϕ has charge $-2\pi/3$.

Below the weak scale the renormalizable effective Lagrangian for the neutrino sector of the theory is

$$\mathcal{L}_\nu^M = g_{ij} \nu_i n_j \phi + \tilde{g}_{ij} n_i n_j \tilde{\phi} + \text{h.c.} + V(\phi, \tilde{\phi}), \quad (2)$$

where $g = \langle h \rangle \lambda / M$, $\tilde{g} = \langle s \rangle \tilde{\lambda}/\tilde{M}$ and the scalar potential is $V = -\mu^2|\phi|^2 + \kappa^2|\phi|^4 - \tilde{\mu}^2|\tilde{\phi}|^2 + \tilde{\kappa}^2|\tilde{\phi}|^4$. (We have assumed that SUSY breaking effects generate *negative* soft mass^2 terms for $\phi, \tilde{\phi}$.) This theory has two accidental $U(1)$ global symmetries: one under which ϕ and ν are charged and another one under which $\tilde{\phi}$, ν and n are charged. (While these symmetries are not exact even at the renormalizable level, the terms that do not respect them are quite small: for example, the ϕ^3 term in $V(\phi)$ is only generated at three loops and can be neglected in the present context.) When ϕ and $\tilde{\phi}$ acquire vevs, these symmetries are broken leading to two pseudo-Goldstone bosons G and $\tilde{G}$, and giving the neutrinos a mass.

With only minor changes we can construct a theory where the six neutrinos are Dirac. There are now three singlet left-handed sterile neutrinos, ν_i^s, and a total of 6 right-handed neutrinos n_α, coupled via

$$\begin{aligned} W^D &= W_{NMSSM} + W_\nu^D, \\ W_\nu^D &= \lambda_{i\alpha} l_i n_\alpha h \, \frac{\phi}{M} + \frac{\kappa}{3}\phi^3 + \tilde{\lambda}_{i\alpha} \nu_i^s n_\alpha s \, \frac{\tilde{\phi}}{M} + \frac{\tilde{\kappa}}{3}\tilde{\phi}^3 \quad (3) \end{aligned}$$

The superpotential W^D is the most general up to dimension four that is invariant under $Z_3 \times Z_3' \times Z_3''$ (with ν^s, like n, having charges $2\pi/3$ under each Z_3) together with a lepton number symmetry under which l and ν^s have the same charge and n the opposite charge.

Below the weak scale the renormalizable effective Lagrangian for the neutrino sector of this theory is

$$\mathcal{L}_\nu^D = g_{i\alpha} \nu_i n_\alpha \phi + \tilde{g}_{i\alpha} \nu_i^s n_\alpha \tilde{\phi} + \text{h.c.} + V(\phi, \tilde{\phi}). \quad (4)$$

The theory has two approximate global symmetries: one under which ϕ and ν are charged, and another under which $\tilde{\phi}$ and ν^s are charged. Again, ϕ and $\tilde{\phi}$ vevs break these symmetries leading to two pseudo-Goldstone bosons and Dirac masses for neutrinos.

Eqs. (2) and (4) imply that g and $\tilde{g}$ are of order m_ν/f, where m_ν is a scale of order the neutrino masses, and f is the scale at which the global symmetries are broken. It is important to note that in general the couplings of the Goldstones to the neutrinos are not diagonal in the neutrino mass basis. Instead, denoting the mass eigenstates by primes, these couplings are of the form $(g_{\alpha\beta}\nu'_\alpha n'_\beta G + \tilde{g}_{\alpha\beta}\nu'_\alpha n'_\beta \tilde{G})$ (Dirac case) and $(g_{\alpha\beta}\nu'_\alpha \nu'_\beta G + \tilde{g}_{\alpha\beta}\nu'_\alpha \nu'_\beta \tilde{G})$ (Majorana case).

These theories provide concrete examples of a very rich set of theories. A particularly simple theory is obtained

by deleting the $\tilde{\phi}$ field and its interactions, and removing the Z_3'' symmetry so that ϕ can couple to both doublet and singlet neutrino mass operators. In this case there is a single flavor diagonal $U(1)$ symmetry and hence a single Goldstone, having diagonal couplings to neutrinos in the mass basis.

Constraints— Significant constraints on the parameter space of these theories follow from the requirement that the total energy density in radiation at the time of BBN does not differ significantly from the Standard Model prediction. This requires that the "hidden sector" fields (ϕ, $\tilde{\phi}$, n and possibly ν^s, as well as the fermionic partners of ϕ and $\tilde{\phi}$ which will turn out to be quite light) not be in thermal equilibrium with the "visible sector" fields $(\nu, \gamma, \ldots)$ before and during the BBN. More precisely, we require that the two sectors decouple at a certain temperature $T_0 > 1$ GeV, and do not recouple until the temperature of the visible sector drops below $T_W \sim 1$ MeV, the temperature at which the weak interactions decouple. If this is the case, the reheating of the visible sector by the decoupling of heavy particles $(\mu, \pi, \ldots)$ and possibly by the QCD phase transition will not affect the hidden sector. Defining r as the ratio of temperatures of the two sectors at the time of BBN, we conclude that the energy density in the hidden sector is suppressed by a factor of r^4 compared to the naive estimate, and $r \lesssim 0.3$ allows one to avoid the BBN constraint even for a very large hidden sector.

The reactions that could recouple the two sectors include a $1 \leftrightarrow 2$ process $\phi \leftrightarrow \nu n$, $2 \leftrightarrow 2$ processes such as $\nu\bar{\nu} \leftrightarrow n\bar{n}$ and $\nu n \leftrightarrow \phi\tilde{\phi}$, $2 \leftrightarrow 3$ processes such as $\nu n \leftrightarrow 3\phi$, etc. Requiring that all these processes be "frozen" $(\Gamma < H)$ prior to the weak interactions decoupling results in the following constraints on the couplings:

$$g_{ij},\;\; g_{i\alpha} \lesssim 10^{-5}, \qquad g_{ij}\kappa,\;\; g_{i\alpha}\kappa \lesssim 10^{-10} r^{-1},$$
$$g_{ij}\tilde{g}_{ij},\;\; g_{i\alpha}\tilde{g}_{i\alpha} \;\lesssim\; 10^{-10} r^{-3/2}. \qquad (5)$$

Note that the coupling $\tilde{\kappa}$ is unconstrained.

The upper bounds on the coupling g can be translated into a lower bound on the symmetry breaking scale f. To interpret the LSND result in the model with Majorana sterile neutrinos, the low-energy theory must possess a mass term of the form $m_{ij}\nu_i n_j$, with at least some elements of m as large as 0.1 eV. This implies that $g_{ij} f \sim 0.1$ eV, and for a generic flavor structure we obtain a bound $f \gtrsim 10$ keV. A similar bound can be obtained for the Dirac sterile neutrino case.

To avoid producing sterile neutrinos by oscillations prior to weak interactions decoupling, we require that the mass terms mixing active and sterile neutrinos not be generated until the temperature of the visible sector drops below T_W. Scattering in the plasma generates "thermal" masses for the ϕ bosons, $m^2(\phi) \sim \kappa^2 n_\phi(T')/T'$, where T' is the temperature of the hidden sector. The symmetry breaking phase transitions for ϕ occurs when $m^2(\phi) \sim \mu^2$. Using $\mu = f\kappa$, we conclude that the temperature of the visible sector at the time of this phase transition is $\sim f/r$, implying that $f \lesssim r$ MeV is necessary for the success of BBN. This in turn imposes a *lower* limit on the couplings, $g_{ij} \gtrsim r^{-1}\, 10^{-7}$.

To summarize, BBN considerations lead to a range of the allowed values of the scale f,

$$10 \text{ keV} \lesssim f \lesssim r \text{ MeV}. \qquad (6)$$

Considerations of the supernova dynamics may slightly raise the lower bound; however, these constraints are strongly model dependent [14]. Even though the allowed values of f are much lower than the weak scale, the theory naturally allows for symmetry breaking in this range. The only assumption necessary is that ϕ only feels supersymmetry breaking through its couplings to l and n. Then μ^2 is of order $g^2 m_{\rm SUSY}^2/16\pi^2$, where $m_{\rm SUSY}$ is a typical soft supersymmetry breaking mass. Since g is of order m_ν/f and f itself is of order μ/κ, by eliminating μ and g in favor of f and m_ν we are led to the expression

$$f \approx \sqrt{\frac{m_\nu\, m_{\rm SUSY}}{4\pi\kappa}} \qquad (7)$$

For reasonable values of the parameters $m_\nu \approx 0.1$ eV, $m_{\rm SUSY} \approx 100$ GeV, $\kappa \approx 10^{-4}$, this yields a value of f of order 3 MeV, which is quite close to the desired range. Analogous considerations apply to the second symmetry breaking scale, $\tilde{f}$.

Interestingly, the large-scale structure limit on the sum of neutrino masses [9] is *automatically* avoided in the models discussed here, and does not lead to additional constraints on f. The lower bound on g_{ij} obtained above implies that the reactions $\nu\bar{\nu} \leftrightarrow n\bar{n}, \phi\bar{\phi}$ become unfrozen *before* the sterile neutrinos become nonrelativistic. These reactions thermalize the hidden sector fields with the active neutrinos. The density of thermal, sterile neutrinos of mass m_s at temperatures $T < m_s$ is suppressed by a Boltzmann factor $e^{-m_s/T}$; the excess neutrinos disappear either via a decay process $n \to \nu\phi$, or via an annihilation process $n\bar{n} \to \nu\bar{\nu}$. As a result, the massive sterile neutrinos do not make a significant contribution to dark matter. It is only the sum of the masses of active, stable neutrinos and the Goldstones that has to satisfy the constraints of Ref. [10].

Signals in the CMB— The non-standard evolution of neutrino masses and densities in our scenario leaves an imprint in the CMB inhomogeneities. There are two distinct, potentially observable effects [12]. First, the total relativistic energy density at the time of last scatter is modified due to the decay and annihilation of the sterile neutrinos. Second, unlike in the standard cosmology, free-streaming of the active neutrinos may be prevented by their interactions with the Goldstone bosons. Let us consider each of these effects.

At the time of BBN, the energy density in the hidden sector is suppressed. When the reactions $\nu\bar{\nu} \leftrightarrow n\bar{n}, \phi\bar{\phi}$,

n_G	Dirac n_s			Majorana n_s		
	1	2	3	1	2	3
2	3.59	3.78	3.95	3.78	3.92	4.06
3	3.70	3.86	4.01	3.91	4.03	4.14
8	4.00	4.11	4.21	4.22	4.29	4.35

TABLE I: Effective number of neutrino species during the recombination era, $N_{\nu,\mathrm{CMB}}$, as determined by the relativistic energy density.

$\nu n \leftrightarrow \phi$ become unfrozen, the two sectors thermalize and the energy density per degree of freedom in the hidden sector approaches that of the active neutrinos. (The active neutrinos themselves are by this time decoupled from electrons and photons.) These reactions, however, do not change the total relativistic energy density: they merely transfer part of the energy from the active neutrinos to the hidden sector states. In contrast, when the sterile neutrinos become non-relativistic ($T \sim m_s \sim 1$ eV) and are depleted by decays and annihilations, the total relativistic energy is increased: the depletion process occurs at constant entropy, resulting in an increase in temperature as the number of relativistic degrees of freedom decreases. Since the depletion of n's occurs before the matter-radiation equality, this will result in a non-standard value of the relativistic energy density implied by the CMB measurements. In terms of the "effective" number of neutrinos $N_{\nu,\mathrm{CMB}}$ [12], our scenario predicts

$$N_{\nu,\mathrm{CMB}} = 3\left(1 + \frac{n_s + 2.75 n_h/g_\nu}{3 + n_G/g_\nu}\right)^{1/3}. \qquad (8)$$

Here, g_ν equals 7/4 for Majorana neutrinos and 7/2 for the Dirac case; n_h is the number of the "Higgs" (massive) components of the scalar fields responsible for global symmetry breaking that are light enough to be relativistic when the reactions $\nu\nu \to \phi\phi$, $\nu n \to \phi\tilde{\phi}$ become unfrozen; n_s is the number of sterile neutrino species, and n_G is the number of Goldstone modes. (Eq. (8) includes the contribution from the superpartners of the symmetry breaking scalar fields.) For example, in the explicit models presented above, $n_s = 3$ and $n_G = n_h = 2$. Some typical values for $N_{\nu,\mathrm{CMB}}$ are presented in Table I. For comparison, while the current sensitivity on $N_{\nu,\mathrm{CMB}}$ from the WMAP and other CMB analyses [15] is about ± 5, the sensitivity of the Planck experiment is expected to reach the ± 0.20 level, providing a test of our predictions.

Furthermore, at the time of last scatter the mean free paths of the light neutrinos and the Goldstones are well below the Hubble scale due to the process $\nu_i \leftrightarrow \nu_j G$. The absence of free-streaming leads to a shift in the positions of the CMB peaks at large l [12, 16]. This shift (relative to the Standard Model prediction) is given by

$$\Delta l_n = 23.3 - 13.1\left(\frac{g_\nu(3 - n_S)}{(3g_\nu + n_G)(1/N_{\nu,\mathrm{CMB}} + .23)}\right) \qquad (9)$$

where n_S is the number of light neutrinos that are scattering during the eV era. (It is possible that $n_S < 3$ if one of the neutrinos is massless or very nearly so, or if m_G is large enough to make the process $\nu_i \to \nu_j G$ kinematically forbidden for some flavors). Eq. (9) provides another experimentally testable prediction of our scenario.

In the theory with no $\tilde{\phi}$ and a single Goldstone, the neutrino decays are absent so that scattering can only occur via the $2 \leftrightarrow 2$ processes $\nu\nu \leftrightarrow GG$ and $\nu G \leftrightarrow \nu G$. In this case the number of neutrino species which scatter is very sensitive to f and to whether the neutrino spectrum is hierarchical, inverted or degenerate.

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